Nonlinear Magnetization in Superconductors with s + d Ordering

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Abstract

The nonlinear magnetization is considered within the Ginzburg-Landau theoretical framework, in the Meissner regime. A calculational method in the case of conventional superconductors (one order parameter) is developed and this method is extended for the case of two order parameters (s+d mixing). It is confirmed that corrections to the penetration depth, in the mean field analysis, are of the order of $H_0{}^2$ where H_0 is the applied field. We analyze carefully the possible solutions which lead to different scenarios in the physics of the symmetry of the order parameter. The anisotropy in the penetration depth is calculated and the temperature dependence of the magnetization is extracted. We discuss the relevant experimental results in the light of these calculations.

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I. INTRODUCTION

The symmetry of the order parameter in high-T_c superconductors is an important and vital question. Knowledge of the symmetry of the superconducting gap function adds information regarding the microscopic mechanisms and reveals novel phenomena as well. An important contribution to this effort comes from one particular experimental technique, the measurement of the penetration depth or the magnetization when magnetic field is applied. The dependence of the penetration depth on the applied field (for small values of the field) and on the temperature is able to provide some conclusions on the order parameter symmetry. The quantity which is most easily measured is the deviation of the penetration depth from its zero-temperture, zero-field value: $\Delta\lambda_{ab}(H,T) = \lambda_{ab}(H,T) - \lambda_{ab}(0,0)$. The indices indicate an in-plane penetration depth. The measurements that have been performed on different materials do not give a completely clear picture. Measurements on $YBa_2Cu_3O_y$ crystals by Sridhar *et al.* [1] result in a quadratic field-dependent penetration depth below H_{c1} and linear dependence above H_{c1}. On the other hand $Bi_2Sr_2CaCu_2O_y$ (BSCCO) shows a quadratic behavior in a region of values of magnetic field much smaller than H_{c1} and linear dependence in the remaining region [2]. The temperature dependence shows a very interesting behavior as well. Hardy *et al.* observed linear T-dependence at low temperatures in YBCO [3] and Bonn *et al.* observed a crossover from linear-T to T^2 upon Zn doping [4]. Also a T^2 behavior has been observed on BSCCO by Maeda *et al.* [5]. The linear field-dependence is predicted by the d-wave scenario, according to Xu *et al.* [6].

At this moment the question of the microscopic mechanism is still lacking a clear consensus, though most work now concentrates on the magnetic mechanism (antiferromagnetic fluctuations in CuO_2 planes) as the first candidate for the d-wave component. Experimental results support an order parameter of predominantly d-wave symmetry, as reviewed in Refs. [7] and [8]. However, there are possible indications of an admixture of s-wave in overdoped and electron-doped materials, as reviewed in Ref. [9]. This latter possibility is consistent with the magnetic hypothesis.

A signature of s-d mixing is the spontaneous appearance of orthorhombic-like anisotropies in the superconducting state. In the penetration depth this would mean $\lambda_x \neq \lambda_y$. In a system such as YBCO with an orthorhombic crystal structure, it is necessary to somehow disentangle this spontaneous anisotropy from the crystalline anisotropy. This paper attempts to do this for one particular measurable quantity, the temperature- and field-dependent penetration depth.

Experiments in YBCO have indeed shown a large difference in the penetration depth along the a axis (λ_a) and b axis (λ_b), which has been attributed to the orthorhombic distortion [10,11]. This distortion can be taken into account in the construction of the free energy. In Ref. [12] the anisotropy at zero temperature has been calculated in a microscopic model where the pairing is of d-wave type and using a single tight-binding band with different hopping parameters in the a and b directions. The point of view is that the chains accomodate a significant part of the condensate and this has to be taken into account. Also in Ref. [13] it was found that the chains cannot become superconducting by proximity effect. The microscopic theory of the non-linear Meissner effect within a purely one-component d-wave scenario has been formulated by Xu *et al.* [6]. Their emphasis was on low temperature behavior, while our Ginzburg-Landau approach is more suited to temperatures neat T_c.

In this paper, the framework of Ginzburg-Landau (G.L.) theory is used and this mean field theory allows us to do calculations taking into consideration the possible symmetry of the order parameter and constructing the appropriate free energy functional. In this manner the specific microscopic mechanism is avoided and the thermodynamic properties can be extracted. The attention is concentrated in the Meissner phase, where the presence of vortices can be neglected and the field can be considered weak. The solution of the G.L. equations beyond the London approximation offers the dependence of the penetration depth and the magnetization on low applied fields and also the temperature. In mean field theory then the dependence on the field is quadratic. The corrections of the order parameter can be calculated analytically as well.

The plan of the paper is as follows. In Sec. II, the G.L. equations are solved in the

case of conventional superconductors in the Meissner phase, so the calculational method is illustrated. We believe it is really worth presenting these calculations, since we were unable to find any detailed calculation in the literature. In Sec. III, the method is extended to the case of the mixed symmetry in the order parameter (s + d). The possible solutions are considered and we discuss the implication of each one. In general the situation where the d-wave is dominant is adopted and the role of the s-wave component is illustrated. This s-symmetry part appears partly because of the perturbation due to the field but a nonzero value even in the absence of external field can be present. The anisotropy in the penetration length along the a and b axes can be calculated easily since the orthorhombic distortion is considered in the development of the free energy functional. In Sec. IV we conclude and discuss the results in connection to relevant experimental data. Also in the Appendix the full set of equations are given and the implication of the different field directions are considered.

II. ONE ORDER PARAMETER - CONVENTIONAL SUPERCONDUCTORS

For the case of the conventional (s-wave) superconductors, the G.L. free energy is written:

$$F = \int \left[\alpha |\Psi(x)|^2 + \beta |\Psi(x)|^4 + \gamma |\vec{\Pi}\Psi|^2 + \frac{h^2}{8\pi} \right] d^3x \tag{1}$$

where $\vec{\Pi} = -i\hbar\vec{\nabla} - \frac{2e}{c}\vec{A}$ and $\vec{B} = \vec{\nabla} \times \vec{A}$. The above free-energy is minimized with respect to Ψ^* and $\vec{A}$. Ψ is written as $\Psi = |\Psi| \, e^{i\phi}$, where ϕ is the phase of the order parameter. Then the equations to be solved are :

$$\gamma \left[-\hbar^2 \vec{\Pi}^2 |\Psi| + (\hbar\vec{\nabla}\phi - \frac{2e}{c}\vec{A})^2 |\Psi| \right] + \alpha\Psi + 2\beta |\Psi|^2 \Psi = 0 \tag{2}$$

$$\frac{c}{4\pi}\vec{\nabla} \times \vec{\nabla} \times \vec{A} = 4\gamma e |\Psi|^2 (\hbar\vec{\nabla}\phi - \frac{2e}{c}\vec{A}) \tag{3}$$

In the London approximation $|\Psi|$ is considered spatially constant, unaltered by the applied field. From Eq. (3) above the penetration depth is found to be $\lambda = \sqrt{(-c^2\beta/16\pi\gamma\alpha e)}$. In the calculations, the applied magnetic field $\vec{H}_0$ is taken parallel to the z-axis, the superconductor occupies the half space $x > 0$ and the most convenient gauge in which to work

is the one with $A_y = A_z = 0$. Then the phase ϕ and the vector potential A_x have x,y- de-pendence only. Since the field and the supercurrent are expected to have only x-dependence (translational invariance in the $\hat{y}$ direction) ϕ and A_x can be written as : $\phi = y\, g(x)$ and $A_x = y\, a(x)$. $|\Psi|$ is expected to have x-dependence as well. After these substitutions (also for convenience $|\Psi| \equiv f$) the above equations become :

$$\gamma\left(-\hbar^2 \frac{d^2 f}{dx^2} + \hbar^2 g^2 f\right) + 2\beta f^3 + \alpha f = 0 \tag{4}$$

$$\frac{dg}{dx} = \frac{2e}{\hbar c}\, a(x) \tag{5}$$

$$\frac{da}{dx} = \frac{8\pi e}{c} f^2 \hbar g \tag{6}$$

Eq. (4)/(5) above is the real/imaginary part of Eq. (2) respectively. The boundary conditions are: $a(x = 0) = H_0$, $a(x = \infty) = 0$, $g'(x = 0) = (\hbar c/2e)H_0$, $g'(x = \infty) = 0$, $f(x = \infty) = f_0 = \sqrt{-\alpha/2\beta}$ and $f'(x = 0) = 0$ [14]. It is obvious then that $g \propto H_0$ and the second term in the l.h.s. of Eq. (4) above serves as a term containing the small parameter of the problem. Then perturbation theory can be used to obtain the solution in small fields (small compared to the thermodynamic field H_c). We consider solutions of the form:

$$f = f_0 + H_0^2\, f_1 + H_0^4\, f_2 + ... \tag{7}$$

In this case, if the above ansatz substitutes f in Eq. (4) and terms with same power in H_0 are gathered ($\tilde{g} \equiv g/H_0$) we get:

$$-\gamma\hbar^2 \frac{d^2 f_0}{dx^2} + 2\beta f_0^3 + \alpha f_0 = 0 \tag{8}$$

$$-\gamma\hbar^2 \frac{d^2 f_1}{dx^2} + 6\beta f_0^2 f_1 + \alpha f_1 = -\gamma\hbar^2 g^2 f_0 \tag{9}$$

$$\frac{\hbar c}{8\pi e} \frac{d^2 \tilde{g}}{dx^2} = \frac{2e\hbar}{c}(f_0^2 + 2H_0^2 f_1 f_2)\, \tilde{g} \tag{10}$$

The last equation comes from the combination of Eqs. (5) and (6). The above expansion is correct, since the equations corresponding to greater powers of H_0 are such that conver-gence is guaranteed. Therefore the above method works and leads to correct results. Then the largest correction due to the field will have an H_0^2 dependence. The solution for f_0 is

the unperturbed part $\sqrt{-\alpha/2\beta}$. The solution for f_1 then can be found easily and the form of the order parameter becomes $(H_c \equiv \frac{\hbar c}{\sqrt{2}\xi\lambda e})$

$$|\Psi| = \sqrt{-\alpha/2\beta}\left\{1 + (\frac{H_0}{H_c})^2 \frac{2}{(\frac{4\xi^2}{\lambda^2} - \frac{1}{2})}\left[-\frac{2\sqrt{2}\xi}{\lambda}\exp(-x/\sqrt{2}\xi) + \exp(-2x/\lambda)\right]\right\}. \qquad (11)$$

An interesting immediate observation is that besides the term with the exponential dependence on the coherence length there is another one which decays with the characteristic length of $\lambda/2$. The above values of ξ and λ are the zero-field ones. In the case of a superconductor with $\xi = \lambda/2\sqrt{2}$ the divergent part in the denominator is cancelled by the numerator. Also, the nonlinear correction f_1 is negative, independent of the type of the superconductor, something which is expected since the magnetic field acts as a pair-breaking mechanism.

The next step is to substitute the above solution in the equation of $\tilde{g}$ and solve it by the same method. We define the effective penetration depth as follows:

$$\lambda_{eff} = \frac{1}{H_0}\int_0^\infty a(x)dx \qquad (12)$$

By performing the integration the effective penetration depth becomes :

$$\lambda_{eff} = \lambda(H=0)[1 + (\frac{H_0}{H_c})^2 \frac{\kappa(\kappa+4\sqrt{2})}{(\kappa+2\sqrt{2})^2}] \qquad (13)$$

The penetration field is an increasing function of the field and has a quadratic dependence on it. It is somewhat remarkable that this expression does not appear anywhere in the published literature, as far as we are aware. It holds for every value of $\kappa = \lambda(H=0)/\xi(H=0)$, and is therefore a generalization of a formula given in Ref. [1]:

$$\delta\lambda/\lambda = 3/4 \times (H_0/H_c)^2, \qquad (14)$$

which holds in the limit $\kappa \to \infty$. The finite κ correction is due to the part of the order parameter that decays with the characteristic length of $\lambda/2$.

III. S+D ORDER PARAMETER SYMMETRY

Recent experimental evidence and also theoretical calculations [15] suggest that a mixed-symmetry order parameter is a possible candidate to explain several features observed in experiments on high-T_c materials in the absence of a field. However, even if there is no s-wave component in zero field, an s-wave component is always formed in the vicinity of vortex cores or due to induced currents or due to surface or impurity effects or, finally, as a result of the orthorhombic distortion in these materials. Within the Ginzburg-Landau theoretical framework, the free-energy functional that takes into account both the symmetry of the material and the order parameter (but with fourth-order derivatives neglected) takes the form [16] :

$$F = \int d^3x \left\{ \alpha_d |\Psi_d|^2 + \beta_2 |\Psi_d|^4 + \gamma_d |\vec{\Pi}\Psi_d|^2 + \alpha_s |\Psi_s|^2 + \beta_1 |\Psi_s|^4 + \gamma_s |\vec{\Pi}\Psi_s|^2 + \beta_3 |\Psi_s|^2 |\Psi_d|^2 \quad (15) \right.$$

$$\left. + \beta_4 (\Psi_s^{*2}\Psi_d^2 + \Psi_d^{*2}\Psi_s^2) + \gamma_v [(\Pi_y \Psi_s)^*(\Pi_y \Psi_d) - (\Pi_x \Psi_s)^*(\Pi_x \Psi_d) + c.c.] + \frac{h^2}{8\pi} \right\}. \quad (16)$$

The orthorhombic distortion has been taken into account, through the mixed-gradient term (and a perturbative bilinear on the two components term). These terms may serve as a "source" for the s-component (it's one of the two "scenaria" that come from the possible solutions). The mixed-gradient term distinguishes a-axis from b-axis as well (c-axis is along the z-direction). The above functional can be used to derive the anisotropy in the penetration depth that is observed. The same G.L. functional has been extensively studied recently for the case of vortices and actually has been derived in the weak-coupling limit for both continuous and lattice Hamiltonians [17–20]. The geometry will be the same (Meissner-geometry) and the difference is that now penetration along both x and y directions have to be considered separately. Another difference comes from the two-dimensional nature of the free energy functional. The orientation of the applied field produces somewhat different results so one has to consider both the "in-plane" case and the case where the field is along the z-direction.

A. London Approximation

After minimization Eq. 16 with respect to $\Psi_s{}^*$, $\Psi_d{}^*$ and $\vec{A}$ the Euler-Lagrange equations to be solved are:

$$(\gamma_d\Pi^2 + \alpha_d)\Psi_d + \gamma_v(\Pi_y{}^2 - \Pi_x{}^2)\Psi_s + 2\beta_2|\Psi_d|^2\Psi_d + \beta_3|\Psi_s|^2\Psi_s + 2\beta_4\Psi_s{}^2\Psi_d{}^* = 0 \qquad (17)$$

$$(\gamma_s\Pi^2 + \alpha_s)\Psi_s + \gamma_v(\Pi_y{}^2 - \Pi_x{}^2)\Psi_d + 2\beta_1|\Psi_s|^2\Psi_s + \beta_3|\Psi_d|^2\Psi_d + 2\beta_4\Psi_d{}^2\Psi_s{}^* = 0 \qquad (18)$$

$$\frac{c}{4\pi}\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \frac{e^*c}{\hbar}\left\{\gamma_d\Psi_d{}^*\vec{\Pi}\Psi_d + \gamma_s\Psi_s{}^*\vec{\Pi}\Psi_d + \gamma_v(\Psi_s{}^*(\hat{y}\Pi_y - \hat{x}\Pi_x)\Psi_d + \right.$$

$$\left. \Psi_d{}^*(\hat{y}\Pi_y - \hat{x}\Pi_x)\Psi_s + c.c.\right\} \qquad (19)$$

From the form of the above two first equations it's easy to verify that the only possibility for the *relative* phase ϕ of the two components is 0 or π, therefore only d±s states are the starting point of our analysis. Physically d+s and d-s are equivalent and the system spontaneously chooses one of these states. The two axes $\hat{x}$ and $\hat{y}$ are identified with the crystallographic a and b in order to make connections with the experiments. The applied field is along the $\hat{z}$-direction and its spatial variation is along the $\hat{x}$-direction. The boundary conditions are:

$$|\Psi_s(x = \infty)| \equiv f_{s0} = const. \qquad |\Psi_d(x = \infty)| \equiv f_{d0} = const. \qquad \vec{A}(x = \infty) = 0$$

$$[\gamma_d\vec{\Pi}\Psi_d + \gamma_v(\hat{y}\Pi_y - \hat{x}\Pi_x)\Psi_s] \cdot \hat{x} = 0 \qquad [\gamma_s\vec{\Pi}\Psi_s + \gamma_v(\hat{y}\Pi_y - \hat{x}\Pi_x)\Psi_d] \cdot \hat{x} = 0$$

$|\Psi_{s0}| \equiv f_{s0}$ and $|\Psi_{d0}| \equiv f_{d0}$ are the bulk values of the two order parameters, without the applied field. If the London approximation is made ($|\Psi_d|, |\Psi_s|$ spatially constant) the penetration depth along the two axes can be found to be:

$$\lambda_x = \left[\frac{c^2}{4\pi e}\frac{1}{(\gamma_d|\Psi_d|^2 + \gamma_s|\Psi_s|^2 + \gamma_v|\Psi_s||\Psi_d|)}\right]^{1/2} \qquad (20)$$

$$\lambda_y = \left[\frac{c^2}{4\pi e}\frac{1}{(\gamma_d|\Psi_d|^2 + \gamma_s|\Psi_s|^2 - \gamma_v|\Psi_s||\Psi_d|)}\right]^{1/2} \qquad (21)$$

The above formulas give a first estimate how it is possible to obtain the difference in the penetration depth from the G.L. theory consistent with the experimental measurements [10]. The ratio λ_x/λ_y is $1 - \epsilon$ where ϵ is the quantity $\frac{\gamma_v}{\gamma_d}\frac{|\Psi_s|}{|\Psi_d|} + O((\gamma_v\frac{|\Psi_s|}{|\Psi_d|})^2)$. In Fig. 1 this ratio

which measures the anisotropy has been plotted as function of temperature and, as we see close to T_s where the s-component grows as we move from the higher temperatures towards the lower ones, the anisotropy becomes stronger. Since the existence of the s-component enhances the anisotropy, close to T_c the anisotropy vanishes.

What this calculation demonstrates is the fact that the anisotropy can shhow a strong signature of s-d mixing if the ratio of the two components is temperature-dependent. The photoemission measurements on BSCCO-2212 [22] have been interpreted as showing just such a dependence [23]. This interpretation can therefore be checked by magnetization measurements, if careful attention is paid to anisotropy.

B. Beyond the London Approximation

For the calculations beyond the London Approximation the method of Sec. II can be followed closely. From the nature of the equations there are two points of view in obtaining the solutions, with substantial difference in the physics of the problem. The first point of view, or the first physical "scenario" is the one starting from the assumption that $\alpha_s \geq 0$, so that the zeroth order (in the magnetic field) value of the magnitude of the $\Psi_s \equiv f_{s0}$ is 0. In other words, the onset of the s-component of the order parameter is caused by the application of the magnetic field and the induced currents and can be viewed as a perturbation to a robust d-wave superconductor, or a small "transformation" from d to d+s, close to the boundary. There are three characteristic length scales in the problem, the two coherence lengths ξ_d and ξ_s that characterize the spatial change of the two distinct order parameters and the penetration depth λ which is the characteristic length of the electromagnetic changes.

The second physical picture is when $\alpha_s < 0$ and a nonzero value of f_{s0} at zero current is considered. Then the superconductivity is truly two-channel and the existence of the s-wave part has to do either with the pairing mechanism or with the departure from the tetragonal symmetry of the lattice and not with the applied magnetic field which at most modifies the form of the order parameter close to the boundary. In this picture there are four different

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characteristic length scales, the two coherence lengths ξ_d, ξ_s and the two penetration depths λ_d and λ_s (combined to one λ) :

$$\frac{1}{\lambda_{x,y}{}^2} = \frac{1}{\lambda_d{}^2} + \frac{1}{\lambda_s{}^2} \pm \frac{\gamma_v}{\sqrt{\gamma_d\,\gamma_s}} \frac{1}{\lambda_d\,\lambda_s} \tag{22}$$

1. Zero $|\Psi_s|$ at zero applied field

In this case there are the following consequences : (i) The onset of s-wave is due to the mixed gradient term and the value of the order parameter $|\Psi_s| \propto \gamma_v \times H_0{}^2$. (ii) Since this mixed gradient term is responsible for the s-wave it is clear that when the y-dependence is examined alone one gets $|d+s|$ state, on the other hand when the x-dependence is examined one gets $|d-s|$. (iii) According to (i) and (ii) anisotropy *cannot* be observed in penetration depth due to term proportional to γ_v in Eq.[15] and [16]. This term finally gets proportional to $\gamma_v{}^2$ and the sign difference cancels out. (iv) The only way to get the anisotropy in penetration depth is to consider different values for the "masses" in different directions. Namely γ_{dx} and γ_{dy} and the same for γ_s possibly. (v) Due to the above observations the quantity λ_x/λ_y is temperature- *independent*.

The two coherence lengths are given by the equations:

$$\xi_s = \sqrt{\frac{\hbar^2 \gamma_s}{-2\alpha_s + 2\beta_3' f_{d0}{}^2}} \qquad \xi_d = \sqrt{-\frac{\hbar^2 \gamma_d}{4\alpha_d}} \tag{23}$$

From these expressions it can be seen that if $\alpha_s = \alpha_{s0}(T - T_s)$ and $\alpha_d = \alpha_{d0}(T - T_d)$ $(T_d > T_s)$ then the actual second order phase transition is at T_d. At T_s there is only a crossover, not accompanied by a phase transition. The coherence length ξ_s doesn't diverge at T_s but at a temperature much closer to T_d.

2. Nonzero $|\Psi_s|$ at zero field

This case can be considered naturally as a consequence of orthorhombic distortion of the lattice. Then the two representations s and $d_{x^2-y^2}$ can be mixed and a bilinear term

in Ψ_d and Ψ_s has to be included in the free energy. The calculations are presented in the Appendix. The main result is that now the penetration depth acquires an anisotropy due to the nonzero value of Ψ_{s0}. The important difference from the case with nonzero Ψ_{s0} is the different temperature dependence which actually can distinguish between the two cases. The distortion of the order parameter is given by:

$$|\Psi_d| = |\Psi_{d0}| + (\frac{H_0}{H_c})^2 \; (A_{1d} \; e^{-x/\sqrt{2}\xi_d} + A_{2d} \; e^{-2x/\lambda} + A_{3d} \; e^{-x/\sqrt{2}\xi_s}) \tag{24}$$

$$|\Psi_s| = |\Psi_{s0}| + (\frac{H_0}{H_c})^2 \; (B_{1s} \; e^{-x/\sqrt{2}\xi_s} + B_{2s} \; e^{-2x/\lambda} + B_{3s} \; e^{-x/\sqrt{2}\xi_d}) \tag{25}$$

The coefficients have been calculated in the Appendix and they depend on the G.L. coefficients (terms beyond the first order in γ_v have been neglected). The thermodynamic field H_c has been taken as $H_c = \frac{\hbar c}{\sqrt{2}e\xi_d\lambda}$. For the $\hat{y}$-direction the results are the same with the substitution $\gamma_v \to -\gamma_v$.

Knowing the corrections of the order parameters, the effective penetration depth can be computed easily (see Appendix). The main point of the calculation is the temperature dependence of the anisotropy λ_x/λ_y. In Fig. 2 the temperature dependence of the ratio of relative corrections due to the field $\frac{\Delta\lambda_x/\lambda_x}{\Delta\lambda_y/\lambda_y}$ is plotted, in the limit of strong type-II supercon- ductivity ($\kappa >> 1$). This quantity is plotted in order to avoid the explicit field-dependence. The relative corrections are larger in the temperature regime where the anisotropy of the penetration depth at zero field is larger. The effect of a small term which measures the orthorhombicity would be to get smooth curves in the region close to T_c.

Again we see that the appearance of the s-wave admixture gives a strong signature in the penetration depth, this time in the nonlinear signal.

IV. DISCUSSION - CONCLUSIONS

The simple picture that this paper offers may be able to give a qualitative point of view of the several features that are observed simultaneously in penetration depth experiments. The basic conclusions are :

(i) The anisotropy in the penetration depth may arise either as a consequence of directional-dependent "masses" or as a consequence of the s and d-wave mixing. As the figures show, mixing may result in strong temperature and field dependence, so the cases can be distinguished. In Ref. [10], considerable anisotropy was observed: $\lambda_a > 1.5\lambda_b$, but the temperature dependence was not measured. If the anisotropy is temperature dependent then the scenario with the non-zero f_{s0} has to be investigated more carefully. It is ineresting that λ_a/λ_b appeared to be strongly sample-dependent in these experiments. While s-d mixing is expected to be a strong function of doping, the effective mass ratio is not.

(ii) The field - dependence of the penetration depth is of order $H_0{}^2$ for both cases at least in the geometry that it is described. This is in fact a consequence of the boundary conditions in the studied geometry.

(iii) The temperature dependence of the anisotropy can be derived explicitly and compared to experiments. Of course the G.L. equations are not valid at low temperatures, therefore nothing can be said for the experimentally observed crossover from T to T^2 dependence of the quantity $1/\lambda(T)^2$ [3].

(iv) Enhancement of the coupling and the transport along the c-axis if the applied field is *in* the $a-b$ plane. Then it's unavoidable to get screening currents along the c-axis and actually the penetration depth has slightly different values from the ones observed when $H_0//\hat{z}$.

(v) Finally the question of d+s versus d-s states arises (and consequently the equivalence of the two directions $\hat{x}$ and $\hat{y}$. In the model that is described through the G.L. equations for a tetragonal crystal structure a spontaneous symmetry breaking is assumed that distinguishes between the two states and directions. The system chooses one of the two equivalent states. Measurements of the effects described here become much more complicated if domains of d+s and d-s form. In YBCO experiments, this requires detwinned crystals.

In Ref. [6] the nonlinear Meissner effect was examined in the framework of a microscopic model. The central mechanism that produces the nonlinearity is the different contributions to the current from the "co-moving" (condensate velocity parallel to Fermi velocity) and "counter-moving" (condensate velocity antiparallel to Fermi velocity) excitations close to

Fermi energy. Then in the case of pure d-wave the correction is linear in the field. But even at very low temperatures this model predicts a transition to a regime where the corrections are of the order $H_0{}^2$. In addition to that a very small admixture of another component is able to change the behavior of the penetration depth at very low temperatures as a function of temperature. Our picture is complementary to this in that it is nonly appropriate for relatively high temperatures. The effects predicted involve relative values of the different gap components which are realistic. The picture has the advantage of being rather general: note that effects such as impurity scattering are included in the parameters of the G.L. functional.

There are several remarks on some other issues of the calculation of the penetration depth. The most important of these is the role of different type of fluctuations. In Ref. [24] has been calculated the existence of Off Diagonal Long Range Order (ODLRO) in the different "states" of the superconductor. It is found that in the Meissner state this phase coherence is destroyed by phase fluctuations below two dimensions. Also only in strongly type II superconductors, the fluctuations of the field can be considered unimportant. In Ref. [25] the critical fluctuations in the penetration depth as the transition is approached from below is studied and it is found that both the penetration depth and the coherence length diverge with the same exponent which has value (0.53) very close to the mean-field one in three dimensions which was obtained in experimental work of Ref. [27] in contrast with the value of $\sim 1/3$ obtained in Ref. [26] on $YBa_2Cu_3O_{6.95}$ pure bicrystals. These issues suggest first that the calculations presented here are valid in a region not very close to T_c since in any case the small parameter of the problem is in fact the applied field in comparison to thermodynamic field which in turn is temperature dependent. Second the fluctuations in the case of two order parameters will have to be included in a subsequent paper because of the sensitivity of the second order parameter which is itself a "secondary" effect.

In summary we have presented the mean-field analysis in the Meissner state of a super-conductor in the case of one and two-component order parameter which is relevant for the high-T_c materials. In the latter case the picture of the onset of the second component due to

different reasons have been presented and several predictions and qualitative explanations have been made.

V. ACKNOWLEDGEMENTS

VI. APPENDIX

We consider the case of the applied magnetic field H_0 parallel to the c-axis and varying along the $\hat{x}$ direction. The geometry is the same as in Part II. By following the same considerations as in Part II the phase can be written as $\phi = y\, g(x)$ and the gauge $\vec{A} = y\, a(x)\hat{x}$. The magnitudes of the order parameters are denoted as $|\Psi_s| \equiv f_s$ and $|\Psi_d| \equiv f_d$. We substitute all the above into the Eqs. (17) and (18). Then by dividing the equations into real and imaginary parts and also by taking into account the power of y, we obtain the independent equations:

$$\frac{dg}{dx} = \frac{2e}{c\hbar}a(x) \tag{26}$$

$$\gamma_d\hbar^2\left(-\frac{d^2 f_d}{dx^2} + g^2 f_d\right) + \alpha_d f_d + \gamma_v\hbar^2\left(\frac{d^2 f_s}{dx^2} + f_s g^2\right) + 2\beta_2 f_d{}^3 + \beta_3' f_s{}^2 f_d = 0 \tag{27}$$

$$\gamma_s\hbar^2\left(-\frac{d^2 f_s}{dx^2} + g^2 f_s\right) + \alpha_s f_s - \gamma_v\hbar^2\left(\frac{d^2 f_d}{dx^2} + g^2 f_d\right) + \beta_3' f_d{}^2 f_s = 0 \tag{28}$$

The magnetic field (which is explicitly contained in g) is considered as the small parameter of the problem, as in the case of the single order parameter and we seek solutions of the form:

$$g = H_0\, \tilde{g} \tag{29}$$

$$f_d = f_{d0} + H_0{}^2\, f_{d1} \tag{30}$$

$$f_s = f_{s0} + H_0{}^2\, f_{s1} \tag{31}$$

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An important point here is that the dependence of f_d and f_s on the field is quadratic because of the boundary conditions at $x = 0$. With different boundary conditions, linear dependence on the field can be obtained. But this is not the case here. Finally, by gathering terms with the same power in the applied field, the equations for f_{d0}, f_{d1}, f_{s0}, f_{s1} are:

$$\alpha_d f_{d0} + 2\beta_2 f_{d0}{}^3 = 0 \qquad (32)$$

$$\alpha_s f_{s0} + 2\beta_1 f_{s0}{}^3 = 0 \qquad (33)$$

$$\gamma_d \hbar^2 (-\frac{d^2 f_{d1}}{dx^2} + \tilde{g}^2 f_{d0}) + \alpha_d f_{d1} + \gamma_v \hbar^2 (\frac{d^2 f_{s1}}{dx^2} + \tilde{g}^2 f_{s0}) + 6\beta_2 f_{d0}{}^2 f_{d1} + \beta_3' f_{s0}{}^2 f_{d1} = 0 \qquad (34)$$

$$\gamma_s \hbar^2 (-\frac{d^2 f_{s1}}{dx^2} + \tilde{g}^2 f_{s0}) + \alpha_s f_{s1} - \gamma_v \hbar^2 (\frac{d^2 f_{d1}}{dx^2} + \tilde{g}^2 f_{d0}) + \beta_3' f_{d0}{}^2 f_{s1} = 0 \qquad (35)$$

The possible solutions of the Eq. (33) are $f_s = 0$ and $f_s = \sqrt{-\frac{\alpha_s}{2\beta_1}}$. In the results presented in the text the difference in the two cases is discussed. The calculations here are presented under the hypothesis of a nonzero f_s. From Eq. (32, 33) the zeroth order value of the two order parameters can be easily obtained. Then the equations (34, 35) suggest the following : It is natural to expect dependence of f_{d1} on the coherence length ξ_d as well as the penetration depth λ (exactly like the case of one order parameter) but due to the appearance of the terms which depend on f_s it is also expected a small dependence on the coherence length ξ_s. The same, by symmetry, is true for the correction of the s-component f_{s1}. So, the homogeneous parts of Eq. (34, 35) have solutions which depend on the corresponding coherence length of each component. Then the non-homogeneous differential equations mix the dependences as discussed above. For the exact form of the solutions, the boundary conditions have to be taken into account. They take the explicit form : $f_{s1}(x = \infty) = f_{d1}(x = \infty) = 0$, $f_s'(x = 0) = f_d'(x = 0) = 0$ and $a(x = 0) = -H_0$, $a(x = \infty) = 0$. Having elaborated the crucial points, it is straightforward, after some lengthy algebra to obtain the full solutions-corrections to the two order parameters. The homogeneous parts of the equations accept solutions of the form : $f_{d1,h} = A_{1d}\, exp(-x/\sqrt{2}\xi_d)$ and $f_{s1,h} = B_{1s}\, exp(-x/\sqrt{2}\xi_s)$. If we substitute these expressions into the homogeneous parts of the differential equations, we obtain the expressions for the two coherence lengths, which are the *same* as in the case of zero f_{s0} up to order γ_v^2. The solutions of the non-

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homogeneous parts are : $f_{d1,inh} = A_{2d}\ exp(-2x/\lambda) + A_{3d}\ exp(-x/\sqrt{2}\xi_s)$ and $f_{s1,inh} = B_{2s}\ exp(-2x/\lambda) + B_{3s}\ exp(-x/\sqrt{2}\xi_d)$. The coefficients can be evaluated by substituting the above expressions into the inhomogeneous parts of the differential equations and by using the boundary conditions. Finally we get :

$$A_{1d} = \frac{4\sqrt{2}}{\xi_d}[-\frac{-\gamma_d f_{d0}(-4/\lambda^2 + 1/2\xi_s{}^2) + \gamma_v f_{s0}(8/\lambda^2 - 1/2\xi_s{}^2)}{\lambda\gamma_d(-4/\lambda^2 + 1/2\xi_d{}^2)(-4/\lambda^2 + 1/2\xi_s{}^2)}$$

$$+ \frac{\lambda\gamma_v f_{s0}}{\xi_s{}^2\gamma_d(1 - (\xi_s/\xi_d)^2)(4/\lambda^2 - 1/2\xi_s{}^2)}] \tag{36}$$

$$A_{2d} = 2\frac{-\gamma_d f_{d0}(-4/\lambda^2 + 1/2\xi_s{}^2) + \gamma_v f_{s0}(8/\lambda^2 - 1/2\xi_s{}^2)}{\xi_d{}^2\gamma_d(-4/\lambda^2 + 1/2\xi_d{}^2)(-4/\lambda^2 + 1/2\xi_s{}^2)} \tag{37}$$

$$A_{3d} = 4\sqrt{2}\frac{\gamma_v\lambda f_{s0}}{\gamma_d\xi_s\xi_d{}^2(1 - (\xi_s/\xi_d)^2)(-4/\lambda^2 + 1/2\xi_s{}^2)} \tag{38}$$

$$B_{1s} = 4\sqrt{2}/\xi_d{}^2[-\frac{\lambda(-\gamma_s f_{s0}(-4/\lambda^2 + 1/2\xi_d{}^2) + \gamma_v f_{d0}(8/\lambda^2 - 1/2\xi_d{}^2))}{\xi_s\gamma_s(-4/\lambda^2 + 1/2\xi_d{}^2)(-4/\lambda^2 + 1/2\xi_s{}^2)}$$

$$+ \frac{\xi_s\gamma_v f_{d0}}{\lambda\gamma_s(1 - (\xi_d/\xi_s)^2)(4/\lambda^2 - 1/2\xi_d{}^2)}] \tag{39}$$

$$B_{2s} = 2\frac{-\gamma_s f_{s0}(-4/\lambda^2 + 1/2\xi_d{}^2) + \gamma_v f_{d0}(8/\lambda^2 - 1/2\xi_d{}^2)}{\xi_d{}^2\gamma_s(-4/\lambda^2 + 1/2\xi_d{}^2)(-4/\lambda^2 + 1/2\xi_s{}^2)} \tag{40}$$

$$B_{3s} = 4\sqrt{2}\frac{\gamma_v f_{d0}}{\gamma_d\xi_s\xi_d(1 - (\xi_s/\xi_d)^2)(-4/\lambda^2 + 1/2\xi_s{}^2)} \tag{41}$$

The effective penetration depth then can be obtained from the solution of the Eq. (19) with the appropriate boundary conditions that have been described, taking into account Eq. (26). The calculation is straightforward and the result is :

$$\lambda_{eff.} = \lambda(1 + (\frac{H_0}{H_c})^2\ (\frac{2C_1}{\lambda/\xi_d + 2\sqrt{2}} + \frac{C_2}{4} + \frac{2C_3}{\lambda/\xi_s + 2\sqrt{2}}) \tag{42}$$

The coefficients C_i, i=1,2,3 above are given by :

$$C_1 = \frac{E}{D}\ A_{1d} + \frac{F}{D}\ B_{3s}$$

$$C_2 = \frac{E}{D}\ A_{2d} + \frac{F}{D}\ B_{2s}$$

$$C_3 = \frac{E}{D}\ A_{3d} + \frac{F}{D}\ B_{1s}$$

where $D = \gamma_d f_{d0}{}^2 + \gamma_s f_{s0}{}^2 + \gamma_v f_{s0} f_{d0}$, $E = 2\gamma_d f_{d0} + \gamma_v f_{s0}$ and $F = 2\gamma_s f_{s0} + \gamma_v f_{d0}$.

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FIGURES

FIG. 1.   The anisotropy λ_x/λ_y at zero field as a function of temperature. The values of the parameters that have been used are $|\Psi_s|/|\Psi_d| = 0.1 \, (\frac{t-0.8}{t-1})^{1/2}$, $\gamma_s/\gamma_d = 1$ and $\gamma_v = 0.6\gamma_d$.

FIG. 2.   The ratio of the relative corrections due to the applied field in the regime of strong type-II superconductor. The values of the parameters are the same as in Fig. 1. In addition to that $\beta_1 = \beta_2 = \beta_3'/2$